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Numerical Solutions for Heat and Mass Transfer of Thermophoretic Magnetohydrodynamic Flow by Using Adomian-Padé Technique

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Abstract

A free convection thermophoretic hydromagnetic flow over a radiate isothermal inclined plate with heat source or heat sink effect is considered. The effects of the thermophoretic parameter and internal heat generation or absorption for both suction and injection cases are discussed. Hence, the transformed boundary layer equation and the new boundary equation are solved by Adomian-Padé technique.

Keywords: Thermophoretic, heat source, Adomian-Padé technique

INTRODUCTION

Thermophoresis has been the subject of abundant studies for many years (Derjaguin, 1965; Rana; Bhargava, 2002). It describes the migration of suspended small micron sized particles in a non-isothermal gas to the direction with decreasing thermal gradient. The velocity acquired by the particle is known as thermophoretic velocity while the force experienced by the suspended particles due to the temperature differences is known as thermophoretic force (Hayat & Qasim, 2010; Kandasamy et al., 2010). The earliest studies on the role of thermopohoresis in laminar flow over a horizontal plate with analysis on cold and hot plate conditions is given by Goren (1977). The study of magnetohydrodynamic (MHD) flow and heat transfer are deemed as of great interest due to the effect of magnetic field on the boundary layer flow control. The effect of suction and injection along a flat plate for free convection have attract interest of many researcher due to the double impacts projected with respect to heat transfer.

The study on MHD flow over an inclined plate with thermophoresis and heat source has never been considered before. In this paper, we will extend the previous on a steady MHD flow with thermophoresis over a permeable radiate inclined plate by Alam et al. (2009) to include the heat source or sink parameter. We also compared the analytical solution by Noor et al. (2012) with employed Adomian decomposition method aided by Padé approximation to treat boundary condition at infinity. The values of skin friction, wall heat transfer, and wall deposition flux are also tabulated.

MATHEMATICAL MODEL

Consider a two-dimensional steady laminar flow of an incompressible electrically conducting fluid over a continuously moving semi-infinite inclined permeable plate with an acute angle α to the vertical. With the $x-$axis measured along the plate, a magnetic field $B(x)$ is applied in the $y-$direction that is normal to the flow direction. Suction or injection is imposed on the permeable plate. The temperature of the surface is held uniform at T_w which is higher than the ambient temperature T_∞. The species concentration at the surface is maintained uniform at $C_w = 0$ while the ambient fluid concentration is assumed to be C_∞. The presence of uniform internal heat source or sink and thermophoresis are considered to study the variation of velocity, heat transfer and concentration deposition on the inclined surface.

Here u and v are the velocity components in the $x-$ and $y-$direction respectively, ν is the kinematic viscosity, g is the acceleration due to the gravity, β is the volumetric coefficient of the thermal expansion, T, T_w and T_∞ are the temperatures of thermal boundary layer fluid, the inclined plate and the free stream respectively, σ is the electrical conductivity, λ_g is the fluid thermal conductivity, ρ is the fluid density, c_p is the specific heat at constant pressure, q_r is the radiative heat flux in the $y-$direction, $Q(x)$ is the internal heating, μ is the dynamic viscosity, D is the molecular diffusivity of the species concentration and V_T is the thermophoretic velocity.

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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)\cos\varphi - \frac{\sigma B^2(x)}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\lambda_g}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{Q(x)}{\rho c_p}(T - T_\infty) + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B^2(x)}{\rho c_p}u^2, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y} - \frac{\partial(V_T C)}{\partial y}. \tag{4}$$

The magnetic induction,

$$B(x) = \frac{B_0}{\sqrt{2x}}. \tag{5}$$

The boundary conditions corresponding for the model are,

$$u = U_0,\ v = v_w(x),\ T = T_w,\ C = C_w \text{ at } y = 0, \tag{6}$$

$$u = 0,\ T = T_\infty,\ C = C_\infty \text{ as } y \to \infty, \tag{7}$$

where U_0 is the uniform plate velocity and $v_w(x)$ represents fluid suction or injection on the porous surface.

The governing equations (2)-(4) can be transformed to a set of nonlinear ordinary differential equations by introducing the following non-dimensional variables.

$$\eta = y\sqrt{\frac{U_0}{2\nu x}},\ \ \varphi = \sqrt{2U_0 \nu x}\, f(\eta),\ \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \ \phi = \frac{C}{C_\infty}, \tag{8}$$

where φ is the stream function that satisfies the continuity equation (1) with

$$u = \frac{\partial \varphi}{\partial y} = U_0 f'(\eta),\ \ u = -\frac{\partial \varphi}{\partial x} = -\sqrt{\frac{U_0 \nu}{2x}}\left[f(\eta) - \eta f'(\eta)\right], \tag{9}$$

Using equations (8) and (9), the following similarity equations with the corresponding boundary conditions are obtained.

$$f''' + ff'' - \gamma\theta\cos\varphi - Mf' = 0, \tag{10}$$

$$(3R + 4)\theta'' + 3RP_r\left(f\theta' + E_c(f'')^2 + E_c M(f')^2 + 2\delta\theta\right) = 0, \tag{11}$$

$$\phi'' + S_c(f - \tau\theta')\phi' - S_c\tau\theta''\phi = 0, \tag{12}$$

subject to,

$$f = f_w,\ f' = 1,\ \theta = 1,\ \phi = 0 \text{ at } \eta = 0, \tag{13}$$

$$f' = 0,\ \theta = 0,\ \phi = 1 \text{ as } \eta \to \infty, \tag{14}$$

where γ is the local buoyancy parameter, Gr_x is the local Grashof number, Re_x is the local Reynolds number, M is the Hartmann number, R is the conduction-radiation parameter, P_r is Prandtl number, E_c is Eckert number, S_c is the Schmidt number, f_w is the permeability of the porous surface with positive value indicates suction while negative value indicates injection and δ is the internal heat source or sink defined respectively.

$$\gamma = \frac{Gr_x}{\mathrm{Re}_x^2},\ \ Gr_x = \frac{g\beta(T_w - T_\infty)(2x)^3}{\nu^2},\ \ \mathrm{Re}_x = \frac{2xU_0}{\nu},\ \ M = \frac{\sigma B_0^2}{\rho U_0}, \tag{15}$$

$$R=\frac{\lambda_g\chi}{4\sigma_1 T_\infty^3},\ \ P_r=\frac{\nu\rho c_p}{\lambda_g},\ \ E_c=\frac{U_0^2}{c_p\left(T_w-T_\infty\right)},\ \ S_c=\frac{\nu}{D}, \tag{16}$$

$$f_w=-v_w(x)\sqrt{\frac{2x}{\nu U_0}},\ \ \delta=\frac{Q_0}{\rho c_p U_0}. \tag{17}$$

SOLUTION APPROACH

In this section, we shall demonstrate the simple application of the Adomian Decomposition Method (ADM) to obtain an approximate solution of equations (10)-(14). First, write equations (10)-(12) in the operator form,

$$L_1 f=Mf'-\gamma\theta\cos\varphi-ff'', \tag{18}$$

$$L_2\theta=\left[\frac{3RP_r}{3R+4}\right]\left(-f\theta'-E_c\left(f''\right)^2-E_cM\left(f'\right)^2-2\delta\theta\right), \tag{19}$$

$$L_3\phi=S_c\left(\tau\theta''\phi+\tau\theta'\phi'-f\phi'\right), \tag{20}$$

where $L_1=d^3/d\eta^3$, $L_2=d^2/d\eta^2$ and $L_3=d^2/d\eta^2$. Applying the inverse operator $L_1^{-1}(.)=\int_0^n\int_0^n\int_0^n(.)\ dt\,dt\,dt$, $L_2^{-1}(.)=\int_0^n\int_0^n(.)\,dt\,dt$ and $L_3^{-1}(.)=\int_0^n\int_0^n(.)\,dt\,dt$ to the left sides of equations (18)-(20) and by employing the boundary condition (13)-(14) gives,

$$f(\eta)=\eta+\frac{\alpha_1}{2}\eta^2+L_1^{-1}[Mf'-\gamma\theta\cos\varphi-ff''], \tag{21}$$

$$\theta(\eta)=1+\eta\alpha_2+\left[\frac{3RP_r}{3R+4}\right]L_2^{-1}\left(-f\theta'-E_c\left(f''\right)^2-E_cM\left(f'\right)^2-2\delta\theta\right), \tag{22}$$

$$\phi(\eta)=\eta\alpha_3+S_cL_3^{-1}\left(\tau\theta''\phi+\tau\theta'\phi'-f\phi'\right), \tag{23}$$

where $\alpha_1=f''(0)$, $\alpha_2=\theta'(0)$ and $\alpha_3=\phi'(0)$ are to be determined. The nonlinear terms in equations (21)-(23) can be decomposed as (Adomian (1994)),

$$ff''=\sum_{k=0}^{\infty}A_k\ ,\ \ f\theta'=\sum_{k=0}^{\infty}B_k\ ,\ \ f''f''=\sum_{k=0}^{\infty}C_k\ ,\ \ f'f'=\sum_{k=0}^{\infty}G_k\ ,\ \ \theta''\phi=\sum_{k=0}^{\infty}H_k, \tag{24}$$

$$\theta'\phi'=\sum_{k=0}^{\infty}J_k\ ,\ \ f\phi'=\sum_{k=0}^{\infty}L_k. \tag{25}$$

Adopting the algorithm for the Adomian polynomials proposed by Zhu et.al (2005), it can be shown that,

$$A_i = \sum_{k=0}^{i} f_k f_k'' , \quad B_i = \sum_{k=0}^{i} f_k \theta_k' , \quad C_i = \sum_{k=0}^{i} f_k'' f_k'' , \quad G_i = \sum_{k=0}^{i} f_k' f_k' , \tag{26}$$

$$H_i = \sum_{k=0}^{i} \theta_k'' \phi_k , \quad J_i = \sum_{k=0}^{i} \theta_k' \phi_k' , \quad L_i = \sum_{k=0}^{i} f_k \phi_k' , \qquad \forall i = 0,..,n. \tag{27}$$

Substituting equations (24)-(25) into equations (21)-(23) and adopting the modified technique of Wazwaz (2006), the simple recursive Adomian algorithm for generating the individual terms of the series solution of equations (10)-(14) are,

$$f_0(\eta) = \eta, \tag{28}$$

$$f_1(\eta) = \frac{1}{2}\alpha_1\eta^2 + L^{-1}\left[Mf_0' - \gamma\theta\cos\beta - A_0\right], \tag{29}$$

$$f_{k+1}(\eta) = L^{-1}\left[Mf_k' - \gamma\theta\cos\beta - A_k\right], \qquad k \geq 1, \tag{30}$$

$$\theta_0(\eta) = 1, \tag{31}$$

$$\theta_1(\eta) = \eta\alpha_2 + \left[\frac{3RP_r}{3R+4}\right]L^{-1}\left[-B_0 - E_c C_0 - E_c M G_0 - 2\delta\theta_0\right], \tag{32}$$

$$\theta_{k+1}(\eta) = \left[\frac{3RP_r}{3R+4}\right]L^{-1}\left[-B_k - E_c C_k - E_c M G_k - 2\delta\theta_k\right], \quad k \geq 1, \tag{33}$$

$$\phi_0(\eta) = \eta\alpha_3, \tag{34}$$

$$\phi_1(\eta) = S_c L^{-1}\left[\tau H_0 + \tau J_0 - L_0\right], \tag{35}$$

$$\phi_{k+1}(\eta) = S_c L^{-1}\left[\tau H_k + \tau J_k - L_k\right], \qquad k \geq 1, \tag{36}$$

The algorithms (28)-(36) are coded using the numerical algebraic package in Maple 15 associate with built in Padé approximants procedure with significant digits 16. To achieve reasonable accuracy we obtain the 20-term approximation of $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$, where the first four terms are given as follows,

$$f_0(\eta) = \eta, \tag{37}$$

$$\theta_0(\eta) = 1, \tag{38}$$

$$\phi_0(\eta) = \eta\alpha_3, \tag{39}$$

$$f_1(\eta) = 1.70393\eta^2 - 1.35999\eta^3 + 0.22596\eta^4, \tag{40}$$

$$\theta_1(\eta) = -0.82300\eta - 1.18170\eta^2 + 0.57652\eta^3, \tag{41}$$

$$\phi_1(\eta) = -0.24690\alpha_3\eta^2 - 0.53204\alpha_3\eta^3 + 0.35213\alpha_3\eta^4, \tag{42}$$

$$f_2(\eta) = 0.17577\eta^5 + 0.09405\eta^6 - 0.09621\eta^7, \tag{43}$$

$$\theta_2(\eta) = 0.07411\eta^4 + 0.00995\eta^5 - 0.04915\eta^6, \tag{44}$$

$$\phi_2(\eta) = 0.21436\alpha_3\eta^5 - 0.18695\alpha_3\eta^6, \tag{45}$$

$$f_3(\eta) = -0.01096\eta^8 + 0.00913\eta^9 + 0.01476\eta^{10}, \tag{46}$$

$$\theta_3(\eta) = 0.05625\eta^7 - 0.00413\eta^8 - 0.00533\eta^9, \tag{47}$$

$$\phi_3(\eta) = -0.09946\alpha_3\eta^7 + 0.12445\alpha_3\eta^6. \tag{48}$$

RESULTS AND DISCUSSION

The value of skin friction $|f''(0)|$ are compared with previous studies (Cortell, 2007; Javed et al., 2011; Noor et al.,2012) and depicted in Table 1. From Table 1, it shows a good agreement with the previous study. Table 2 showed the comparison between different order of Padé approximants for several f_w. It was found that, the skin friction is higher for the case of injection $f_w = -0.5$ on the inclined permeable surface. However, the heat transfer from the surface and the deposition flux are greater for the suction when $f_w = 0.5$. For Table 3, also showed the comparison between different order of Padé approximants for several value of δ. Here $\delta = -1$ represents heat sink, $\delta = 0$ is without heat source or sink whereas $\delta = 1$ represents heat source. From these tables, it is clearly demonstrate that the convergence of solution are better for higher order of approximations.

Table 1: The comparison of $|f''(0)|$ value for $\gamma = M = f_w = 0$

Cortell (2007)	Javed et.al. (2011)	Noor et.al. (2012)	ADM-Padé (Present)
0.6275	0.6275	0.6275	0.6275

Table 2: Numerical values of $f''(0)$, $-\theta'(0)$ and $\phi'(0)$ for $\gamma = 10, P_r = 0.7, M = 0.5,$ $R = 1, \beta = \pi/6, E_c = 0.1, S_c = 0.6, \tau = 1$ and $\delta = 1$

	f_w	$[5,5]$	$[6,6]$	$[7,7]$	$[8,8]$
$f''(0)$	-0.5	3.39923	3.39924	3.39924	3.39924
	0	3.40785	3.40786	3.40786	3.40786
	0.5	3.31518	3.31519	3.31519	3.31519
$-\theta'(0)$	-0.5	0.73944	0.73945	0.73945	0.73945
	0	0.82301	0.82300	0.82300	0.82300
	0.5	0.91258	0.91259	0.91259	0.91259
$\phi'(0)$	-0.5	0.64332	0.64333	0.64333	0.64333
	0	0.79925	0.79926	0.79926	0.79926
	0.5	0.97276	0.97277	0.97277	0.97277

Table 3: Numerical values of $f''(0)$, $-\theta'(0)$ and $\phi'(0)$ for $\gamma = 10, P_r = 0.7, M = 0.5,$ $R = 1, \beta = \pi/6, E_c = 0.1, S_c = 0.6, \tau = 1$ and $f_w = 0.5$

	δ	$[5,5]$	$[6,6]$	$[7,7]$	$[8,8]$
$f''(0)$	-1	6.51395	6.51396	6.51396	6.51396
	0	4.28481	4.28480	4.28480	4.28480
	1	3.31518	3.31519	3.31519	3.31519
$-\theta'(0)$	-1	0.47728	0.47729	0.47729	0.47729
	0	0.46833	0.46834	0.46834	0.46834
	1	0.91258	0.91259	0.91259	0.91259
$\phi'(0)$	-1	1.16398	1.16399	1.16399	1.16399
	0	1.05036	1.05037	1.05037	1.05037
	1	0.97276	0.97277	0.97277	0.97277

CONCLUSIONS

The effects of thermophoretic and heat source or sink parameters for both suction and injection cases on MHD flow over an inclined radiate isothermal permeable surfaces have been studied. The results obtain in this study suggest the ADM-Padé approach as one of the simplest convincing tool for solving more complex boundary layer conditions numerically in the future.

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