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Effect of Support Domain on Radial Point Interpolation Method (RPIM) for Displacement Analysis in 2D Problem

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Abstract: The Radial Point Interpolation Method (RPIM) is one of the Meshfree methods. RPIM approximation function passes through each node point in the influence domain, thus makes the implementation of essential boundary conditions much easier and reducing complexity in numerical algorithms than other Meshfree methods. However, without the use of predefined mesh, there will be considerable differences in the location of the nodes thus causing topological errors. This topological error will create unstable solutions for the simultaneous equations. This present study is concerned with developing a more efficient solution by introducing a support domain in the RPIM. The study is to outline the complete procedures for formulations of RPIM with support domain for two-dimensional plane stress problems and write the corresponding MATLAB source code. The performance of the optimum size of the support domain is evaluated then compare to Finite Element Method (FEM). The result shows that RPIM with support domain, works well and provides an approving comparison against the conventional FEM. The converged solution is achieved.

Keywords: Numerical Analysis; Meshfree Method; Radial Point Interpolation Method; Support Domain.

1. Introduction

Simulation-based design in engineering is becoming very important nowadays due to the advancement of computing technology. It changes the way engineers interact with engineering problems significantly. Engineers use software to design and analyse engineering problems, thereby allowing engineers to be able to deal with various problems in less time with accurate solutions (Shaikh, 2012). In this area, the Finite Element Method (FEM) is a key component that has been used in the analysis. However, the creation of a mesh in FEM has been much discussed in terms of programming capacity, leading to the development of a new numerical technique without the need for mesh creation, *i.e.*, the Meshfree method.

The Meshfree method has recently risen to prominence as one of the most important methods in numerical analysis. The Meshfree methods are identical to FEM except for the construction of the shape functions, which eliminates the necessity for the mesh. The shape functions are built for a specific point of interest without the need for element-based interpolation. This idea eliminates the requirement for elements as well as the assembly procedure. Meshfree is a family of techniques that attracted the attention of many researchers in their studies (Nayroles *et. al*. 1992; Belytscho *et. al*. 1994; Liu *et. al*. 1995; Liu *et. al*. 1997; Atluri *et. al*. 1998; Mokhtaram *et. al*. 2020).

The Radial Point Interpolation Method (RPIM) is one of the Meshfree methods (Zahiri, 2011). Its approximation function passes through each node point in the influence domain, thus makes the implementation of essential boundary conditions much easier and reducing complexity in numerical algorithms than other Meshfree methods (Wang *et al*. 2002). However, without the use of predefined mesh, there will be considerable differences in the location of the nodes thus causing topological errors. The topological error will create unstable solutions of simultaneous equations. To overcome these problems, support domain is introduced to the domain problem (Martinez, 2019).

This present study concerns with developing a more efficient technique by introducing support domain in the RPIM for 2D plane stress analysis. The study is to outline the complete procedures for formulations of RPIM with and without support domain and write corresponding MATLAB source code and evaluate the performance of the optimum size of support domain then compare to established numerical analysis, i.e., FEM (Liu and Gu, 2005).

2. Research Methodology

The general procedure of the research methodology can be described by the following diagram as shown in Figure 2.1. The figure shows the procedure in a step-by-step manner on the formulations of RPIM with support domain for two-dimensional plane stress problems.

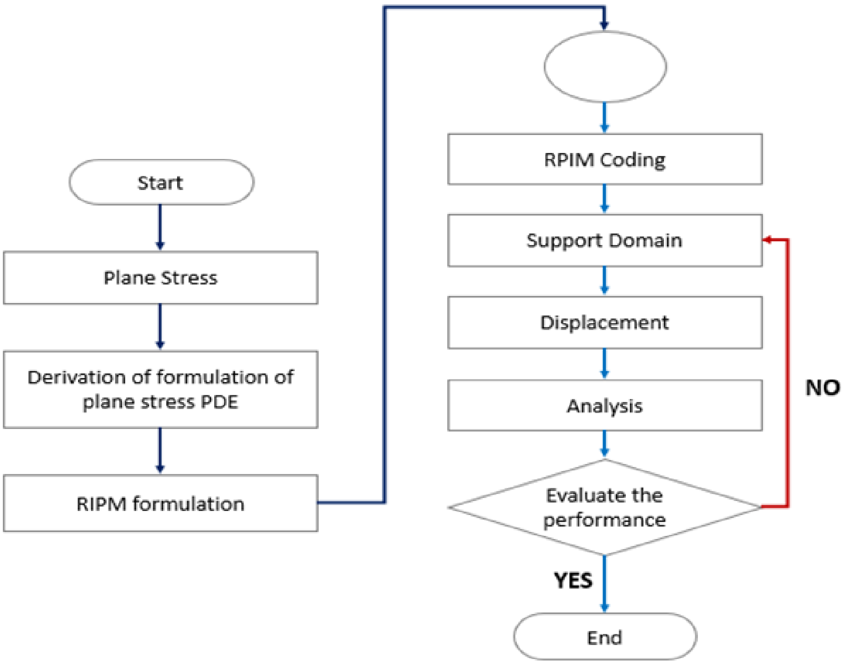

Figure 2.1: Flowchart of RPIM with support domain for two-dimensional plane stress problems

2.1 Radial Basis Function (RBF)

Radial Basis Function (RBF) is used to develop the RPIM shape functions. Consider a function $u(\boldsymbol{x})$ defined in the problem domain with a number of scattered field nodes. For the point of interest x_Q the field function $u(\boldsymbol{x})$ is first approximated using RBFs as follows;

$$u(\boldsymbol{x}) = \sum_{i=1}^{n} R_i(\boldsymbol{x})a_i + \sum_{j=1}^{m} P_j(\boldsymbol{x})b_j = \boldsymbol{R}^T(\boldsymbol{x})\boldsymbol{a} + \boldsymbol{P}^T(\boldsymbol{x})\boldsymbol{b} \quad (1)$$

where i and j are the running indexes, $R_i(\boldsymbol{x})$ is a RBF and $P_j(\boldsymbol{x})$ is a polynomials in the space coordinates $\boldsymbol{x} = (x, y)$, n is the number of RBFs and m is the number of polynomial basis functions. When $m = 0$, pure RBFs are used. Otherwise, the RBF is augmented with m polynomial basis functions. a_i and b_j are constants yet to be determined.

RBFs come in a variety of forms (Wendland, 1998). This study employs RBF Multi-quadrics (RBF-MQ) and RBF Thin plate spline (RBF-TPS), given respectively as;

$$R_i(\boldsymbol{x}) = (r_i{}^2 + (\alpha_c d_c)^2)^q \quad (2)$$

$$R_i(\boldsymbol{x}) = r_i{}^\eta \quad (3)$$

where α_c, q, and η are dimensionless shape parameters, and d_c is the characteristic length. r_i is the distance between $\boldsymbol{x_Q}$, and a node at $\boldsymbol{x_i}$, defined as;

$$r_i = \sqrt{(x_Q - x_i)^2 + (y_Q - y_i)^2} \quad (4)$$

The coefficients a_i and b_j can be determined by enforcing Eqn. (1) to be satisfied at the nodes in the domain. This is done by evaluating the equation subsequently at each node point. This leads to n linear equations, one for each node. Due to the addition of m terms contributed by the polynomial basis functions, additional equations are needed to obtain a unique solution. Therefore, the additional requirement can be applied to the system as;

$$\sum_{j=1}^{n} P_j(x) a_i = \boldsymbol{P}_m{}^T \boldsymbol{a} = 0 \quad (5)$$

the field variables thus can be given as;

$$u(\boldsymbol{x}) = \{\boldsymbol{R}^T(\boldsymbol{x}) \quad \boldsymbol{P}^T(\boldsymbol{x})\}\boldsymbol{G}^{-1}\widetilde{\boldsymbol{U}} = \boldsymbol{\Phi}(\boldsymbol{x})\widetilde{\boldsymbol{U}} \quad (6)$$

where $\boldsymbol{\Phi}(\boldsymbol{x})$ are the RPIM shape functions.

2.2 Plane Stress Formulation

For completion, the plane stress formulation is detailed in this section. For a domain Ω bounded by a boundary Γ, the state of equilibrium, is given as;

$$\boldsymbol{L}^T\boldsymbol{\sigma} + \boldsymbol{b} = 0 \quad in\,\Omega \quad (7)$$

where $\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{L}$ is a matrix of differential operators, and $\boldsymbol{b}$ is the body force of the domain Ω. The standard Galerkin weak form of the Eqn. (7) can be written in matrix forms as;

$$\iint [N]^T[\partial][E][\partial]^T[N]\{\hat{u}\}^T\,dy\,dx = \iint [N]^T\{f\}\,dy\,dx \int [N]^T\{b\}^T ds \quad (8)$$

The left-hand side terms of Eqn. (8) is the stiffness matrix [k] and the right-hand side of the equation is the vector force {f}.

$$[k] = \iint [N]^T [\partial][E][\partial]^T [N]\, dy\, dx \tag{9}$$

$$\{f\} = \iint [N]^T \{f\}\, dy\, dx \int [N]^T \{b\}^T ds \tag{10}$$

Alternatively, in matrix form, the discretized equation of the problem can be represented as given in Eqn. (2.5).

$$[k]\{\hat{u}\}^T = \{f\} \tag{11}$$

2.3 Support Domain

The domain will be discretized using randomly located points, which will be associated with a support domain with the surrounding nodes. These domains will be constructed with simple geometric forms and from their contributions it will be possible to construct the approximation on the general domain. In generating the support domain, it should be based on the number of nodes in the basis function to meet the Kronecker delta condition as shown in Figure 2.2.

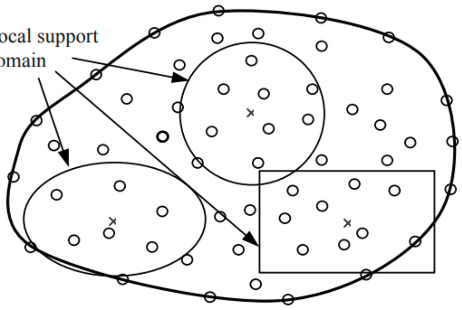

Figure 2.2: Support domains

3. Numerical Example

Here, numerical examples are presented to evaluate the performance of the developed formulatiom for the analysis of two-dimensional plane stress problems. Cantilever beam subject to end loading is chosen to examine the ability of the RPIM with support domain in dealing with structure mechanics.

The geometry and the material properties of the beam are depth D = 0.5 m, length L = 8 m, Young's modulus E = 205 kPa and Poisson's ratio υ = 0.25. The cantilever beam is subjected to end loading 35 kN as shown in Figure 3.1.

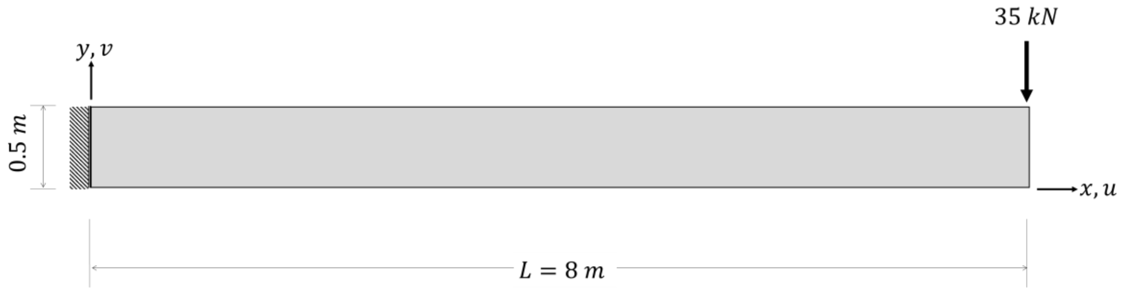

Figure 3.1: A cantilever beam subject to end loading

The "exact" solution herein is defined from a FEM commercial software that is COMSOL, with a very fine mesh. Table 3.1 shows the locations of displacement result and the exact solution.

Table 3.1. The locations of displacement result and the exact solution

Location (x,y)	Displacement (FEM)	
	X-displacement	Y-displacement
(8,0)	-0.013096	-0.280229
(6,0.25)	0	-0.177393
(4,0)	-0.009827	-0.087722
(4,0.25)	0	-0.087655
(4,0.5)	0.009827	-0.087722

The choice of size for support domain depends on the number of nodes. Therefore, to understand the behavior of the support domain, three models have been used in this study based on the total number of nodes and the number of Gauss points as shown in Table 3.2.

Table 3.2. The total number of nodes and the number of Gauss points

	Dirextion of nodes		Number of nodes	Number of Gauss Points
	N_x	N_y		
Model 1	300	10	3311	300
Model 2	350	12	4563	300
Model 2	450	14	6765	300

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An increase in the number of nodes will increase the number of support domains and in turn increase the diameter of the support domain. This will allow the size of the matrix as a whole to be controlled. The support domain used to reduce the size of the matrix are 2.7m, 2m and 1m for Models 1, 2 and 3, respectively as shown in Figure 3.2.

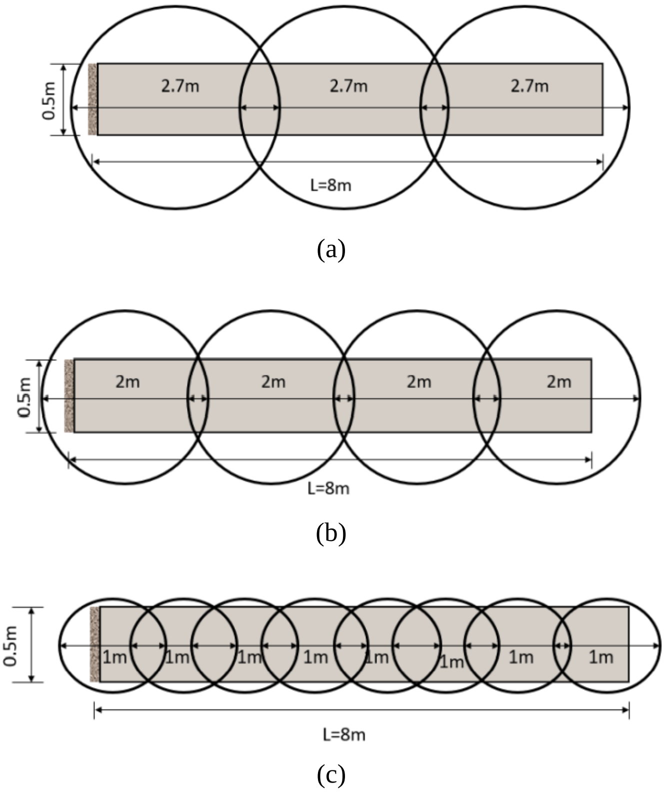

Figure 3.2: Support domains for (a) Model 1, (b) Model 2 and (c) Model 3

4. Results and Discussion

The displacement result of the RPIM with and without support domains is compared with exact solution *i.e.*, COMSOL software, from Table 3.1. Table 4.1 demonstrates the percentage of displacement error in the x and y directions of the problem.

Table 4.1. Prcentage error for Models 1, 2 and 3

Location (x,y)	Direction of disp.	Displacement error (%)					
		Model 1		Model 2		Model 3	
		without support domain	*with support domain*	*without support domain*	*with support domain*	*without support domain*	*with support domain*
(8,0)	*u*	23.50	0.50	31.13	0.37	46.40	0.11

	v	17.89	0.78	28.59	0.28	64.28	0.10
(6,0.25)	u	0.0	0.0	0.0	0.0	0.0	0.0
	v	14.38	0.89	14.72	0.21	71.09	0.08
(4,0)	u	17.62	0.63	27.80	0.10	37.98	0.09
	v	3.03	1.12	4.76	0.11	73.16	0.02
(4,0.25)	u	0.0	0.0	0.0	0.0	0.0	0.0
	v	3.82	1.12	3.96	0.00	38.19	0.03
(4,0.5)	u	17.64	0.63	27.81	0.10	37.99	0.09
	v	3.03	1.12	4.76	0.11	73.16	0.02

Table 4.1 shows that, high displacement error were obtained for all three models for RPIM analysis without support domains. However, good agreement was achieved when the support domain was employed in the formulation. This shows that the topological error from the problems has been solved by reducing the number of equations to be solved by introducing the support domain. The results also show that the size of each support domain will depend on the number of nodes used. This has been shown from comparisons over the three models. The high number of nodes for Model 3 results in more support domains required than Models 1 and 2. Such verifications give a confirmed level of confidence and validate the use of the support domain in the analysis of RPIM.

5. Conclusion

In this paper, the concept of support domains on the analysis of the RPIM is presented and discussed in a step-by-step manner. The RPIM method is constructed based on a Galerkin formulation with the adoption of RBF to produce the shape functions. The method was applied for 2D plane stress problem. Verifications of the problem has been compared with conventional RPIM without support domains and FEM with a very fine mesh. The developed method improves on the existing techniques in the following ways. Without the use of predefined mesh in Meshfree methods, there will be considerable differences in the location of the nodes thus causing topological errors. This situation will become more critical when a large number of nodes are used to obtain converged results. This will create unstable solutions of the simultaneous equations. The use of the support domain has reduced differences in the location of the nodes and in turn, has managed to provide good results.

6. References

Atluri, S. N. and Zhu, T. 1998. A new meshless local Petrov-Galerkin (MLPG) approach in computational mechanics. *Computational Mechanics*, 22, 117-127.

Belytscho, T., Lu, Y.Y. and Gu, L. 1994. Element-free Galerkin methods. *Int. J. Numerical*. Methods. Engrg, 37, 229-256.

Liu, G.R., and Gu, Y.T. 2005. An introduction to meshfree methods and their programming.

Liu, W. K., Jun, S., Li, S., and Zhang, Y.F. 1995. Reproducing kernel particle methods. *Int. J. Numer. Meth. Eng.*, 20, 1081-1160.

Liu, W. K., Li, S. F., and Belytschko, T. 1997. Moving least-square reproducing kernel methods (I) Methodology and convergence. *Comput. Meth. Appl. Mech. Eng*., 143, 113–154.

Martinez, J. 2019. Element Free Galerkin (EFG) sensitivity study in structural analysis. *Research, Innovation and Development in Engineering*. 519 (2019) 012004.

Mokhtaram, M. H., Mohd Noor, M. A., Jamil Abd Nazir, M. Z., Zainal Abidin, A. R., and Mohd Yassin, A. Y. 2020. Enhanced meshfree RPIM with NURBS basis function for analysis of irregular boundary domain. *Malaysian Journal of Civil Engineering*. 32:1 (2020) 19–27.

Nayroles, B., Touzot, G. and Villon, P. 1992. Generalizing the finite element method: diffuse approximation and diffuse elements. *Computational Mechanics*, 10, 307-318.

Shaikh, F.U.A. 2012. Role of commercial software in teaching finite element analysis at undergraduate level: A case study. *Eng. Educ*., vol. 7, no. 2, pp. 2–6.

Wang, J.G., and Liu, G.R. 2002. A point interpolation meshless method based on radial basis functions. *Int. J. Numer. Methods Eng*., vol. 54, no. 11, pp. 1623–1648.

Wendland, H. 1998. Error estimates for interpolation by compactly supported radial basis functions of minimal degree. *J Approximation Theory*, 93, 258–396.

Zahiri. S. 2011. Meshfree Methods, in Numerical Analysis - Theory and Application, *InTech*.