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Selangor Science &Technology Review
Special Issue: Transformation of Manufacturing in Digital Age
Vol. 5, No. 1, (2021)

Non-Linear System Controller Design Using Sliding Mode Control

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Abstract: Throughout the years, many researchers have problems in stabilizing non-linear, uncertain system. One of the methods used to design a robust state feedback controller for an uncertain system is variable structure control (VSC). VSC systems are designed to drive the system states to a sliding surface in the state space in sliding mode control. By using a feasible high-speed switching feedback control to achieve the desired plant behaviour or reaction, the controller structure around the plant is deliberately modified. The problem of developing a VSC law for uncertain system, Electrohydraulic control system is considered in this paper. An alternative design method of a linear sliding surface that is linear to the state is generated using the LMI technique. A proper condition is given for the linear sliding surface to exist. In addition, an explicit linear sliding surface formula to ensure quadratic stability is derived from the reduced-order equivalent system dynamics, constrained to the sliding surfaces. Then, an Electrohydraulic servo system is applied to the sliding mode controller. MATLAB/SIMULINK software was used to carry out the simulation work. Comparison of the controller design, using LMI approach method with and without the sliding mode control shows that LMI approach with a sliding mode control method produces a better performance response.

Keywords: Electrohydraulic system, LMI approach, Sliding mode control, Uncertain system, VSC (Variable structure control)

1. Introduction

The Electrohydraulic control system is a complex system with regard to non-linearity. The linearization-based method has been suggested as an effective way of using the non-linear model of the system in the control law. However, the linearized model approximates the real system dynamics. The latter having uncertainties, the sliding mode controller (SMC) is then preferred because of its robust character and superior performance.

SMC design involves two crucial steps; the first phase is to design a set of sliding manifolds so that the system state restricted to them has desired dynamics, which is of lower order than the original systems. The second phase is to design switching feedback control so that the system state trajectories can be attracted to the designed sliding manifold in finite time and maintain on the manifold (Spurgeon and Edwards, 1998). By applying the proposed controller, the perturbed sliding surface equation is enforced to zero, and by an appropriate choice of this surface, the tracking error tends asymptotically to zero in finite time and with no chattering problems.

2. Methodology

The method used to complete this project is according to the following stages:

(a) Decompose the complete model of the Electrohydraulic system complete model into an uncertain model with nominal values and limited uncertainties (Rong-Fong Fung, 1997).

(b) Determine the dynamics of the system during Sliding Mode.

(c) Design the Sliding Mode Controller for the uncertain system based on LMI approach (Xiang, Su and Chu, 2005).

2.1 System Description

Consider the following uncertain system that can be expressed by the following dynamic equation:

$$\dot{x}(t) = (A + \Delta A(x,t))x(t) + Bu(t) + F(\omega,t) \quad (1)$$

in which

$x(t)$: system state,

$u(t)$: control input,

A : system characteristic matrix,

B : input matrix with full rank m,

$\Delta A(x,t)$: system's uncertainties,

$F(\omega,t)$: system's non-linearity and disturbances.

And it is assumed that the following conditions are valid.

C1: Matrix pair (A, B) is controllable.

C2: State of x is available.

C3: B matrix is full rank, where m<n.

C4: ΔA, and F are continuous on x and piecewise continuous on t.

C5: The matching condition is met and the functions.

$H(p,t): \Re \times \Re^n \to \Re^{m \times n}$ and $E(p,t): \Re \times \Re^n \to \Re^m$ exist, such that,

$$F(\omega,t) = BE(\omega,t), \Delta A(\omega,t) = BH(\omega,t)$$

Thus, the uncertainties can be lumped out of these assumptions, and the system (1) can be rewritten as:

$$\dot{x}(t) = Ax(t) + B[u(t) + g(\omega,t)] \quad (2)$$

in which the $g(\omega,t)$ is the lumped uncertainties. Note that the system varies with the time-varying parameter, ω. Thus, the following condition can be applied if the bound is known.

C6: Positive scalar valued function, $\rho(\omega,t): \Re \times \Re \to \Re_+$ exists, such that $\|g(\omega,t)\| \leq \rho(\omega,t)$.

2.2 Design of Sliding Surface

The system nominal can be partitioned and presented as below:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ B_1 \end{bmatrix} u(t) \tag{3}$$

Using the system (3), the Electrohydraulic control system model, then becomes:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_{31} & -a_{32} & -a_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_{31} \end{bmatrix} u(t) \tag{4}$$

Take the 3 x 3 matrix of (4.4) and partition it into a 2 x 2 matrix as below:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_{31} \end{bmatrix} u(t) \tag{5}$$

Where the sub matrix blocks are defined as:

$$X_1 = \begin{bmatrix} X_1 & X_2 \end{bmatrix}$$
$$X_2 = \begin{bmatrix} X_3 \end{bmatrix}$$
$$A_{11} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$
$$A_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$A_{21} = \begin{bmatrix} -a_{31} & -a_{32} \end{bmatrix}$$
$$A_{22} = \begin{bmatrix} -a_{33} \end{bmatrix}$$

The sliding surface is described as:

$$\sigma(t) = \{X \in \Re^n : SX(t) = 0\} \tag{6}$$

In which $S = [S_1 \quad S_2]$ and $S \in \Re^{m \times n}$, is a full rank matrix.

During the ideal sliding of the surface, the motion can be described as:

$$S_1X_1 + S_2X_2 = 0$$

or $$X_2 = -\frac{S_1}{S_2}X_1 = -S_2^{-1}S_1X_1 = -MX_1 \quad (7)$$

From system in (5):

$$\dot{X}_1 = A_{11}X_1 + A_{12}X_2 \quad (8)$$

Substitute (7) into (8) and by assuming $S_2=I$, the equation (8) can be rewritten as:

$$\dot{X}_1 = (A_{11} - A_{12}M)X_1 \quad (9)$$

For the system to be stable, the value of S_1 must be calculated such that $(A_{11} - A_{12}M)$ is negative definite. This S_1 can be calculated by using the LMI method.

After re-casting the problem using LMI approach, the following equation is obtained:

$$AQ + QA^T - BY - Y^TB^T < 0 \quad (10)$$

The equation (10) is now linear in terms of LMI. Using MATLAB/ LMI Toolbox, *Y* and *Q*'s value can be calculated; thus, S_1 can be found.

The LMI equation (10) is defined and set-up using LMI toolbox,

Where

$$Y = MQ$$
$$Q = P^{-1}$$
$$M = S_2^{-1}S_1 = I^{-1}S_1 = S_1 \qquad and \quad Q > 0$$

Before the value of S_1 can be found, the best value of *t* should be negative for feasibility. Here, t_{min} obtained is:

t_{min} = -6.5552e+004 s

Thus, the value of gain matrix K can be obtained, and the value is:

K =1.0e+004 * [-0.0008 -3.8697 -0.2192]

And the sliding matrix S, is: S = [1.3125 0.9375 1.0000]

3. Results and Discussion

The first step in the design of Sliding Mode Control (SMC) is to parameterize the sliding surface in such a way that the system restricted to the sliding surface exhibits the desired system behaviour. As covered by (Decarlo et al., 1998), under the Sliding Mode Control, the order of the system will be reduced once the system slides on the designed surface. This can be seen by treating $g(\omega,t) = 0$ in (2). The system equation for the Electrohydraulic control system model is given in (4).

3.1 Simulation of Sliding Mode Controller Design using LMI Method

The obtained matrices, K and S values, and the controller, were used and applied on the SIMULINK model as in Fig. 1.

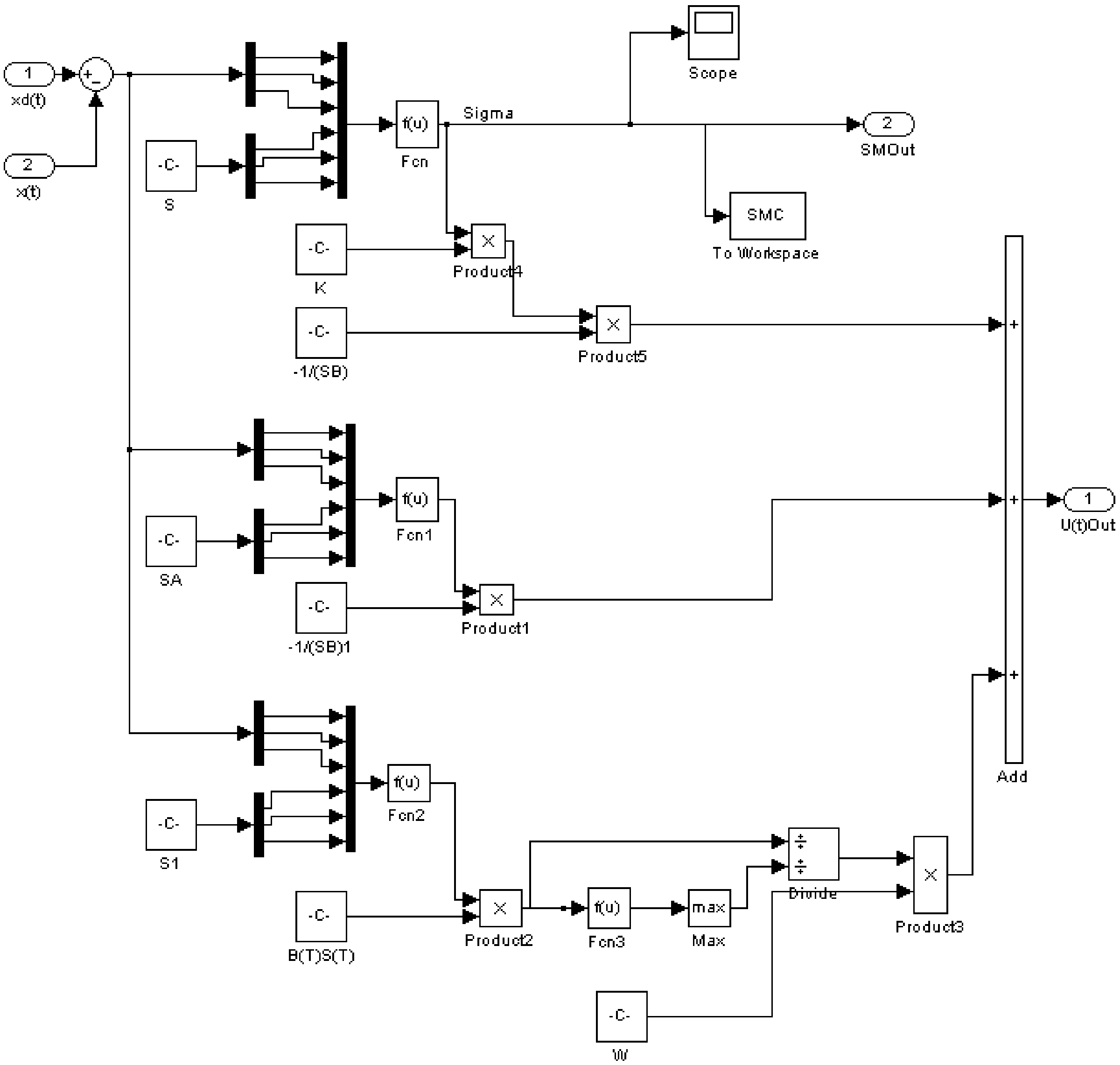

Fig. 1. Simulink Model for SMC Design using LMI Method.

The resulting responses are observed for the motor shaft's angular position, as shown in Fig. 2. The graph shows that the actual responses follow the desired response quite closely, with minimal error.

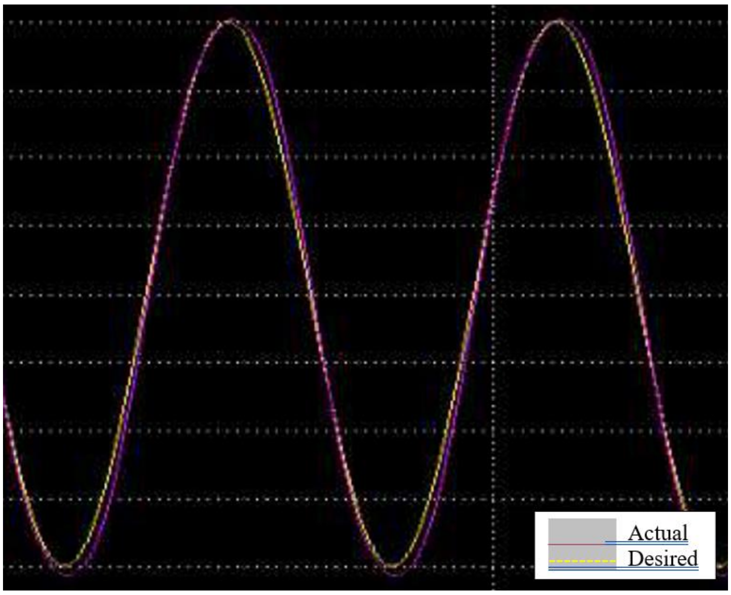

Fig. 2. Actual and Desired Angular Position Responses vs Time

In Fig. 3, the unit step input responses for different controller design methods are compared and shown that the sliding mode design using the LMI method gives the best responses.

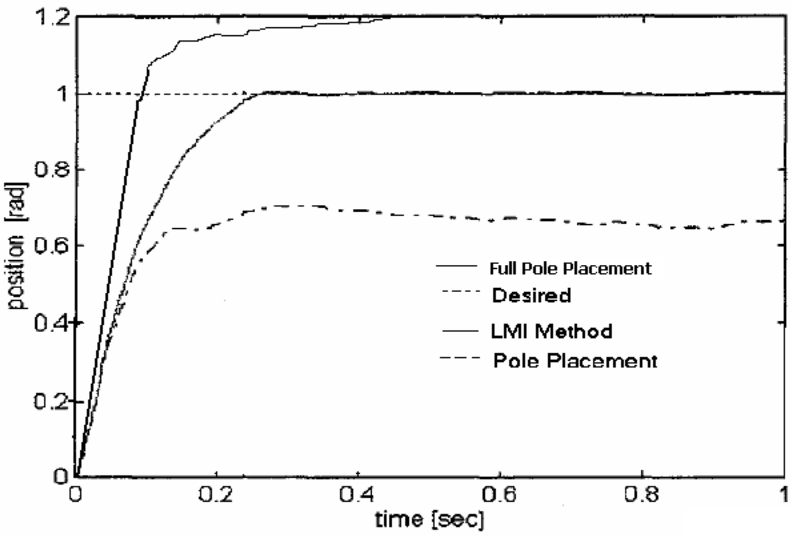

Fig. 3. Angular Position for a Unit Step Input for Different Method

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4. Conclusion

Two key stages are involved in the Sliding Mode Control design. The first stage is to design a series of sliding manifolds so that the system state restricted to them has desired dynamics. The second stage is to design switching feedback control such that the trajectories of the system state can be attracted in finite time to the designed sliding manifold and maintain on the manifold (Choi, 1998). The perturbed sliding surface equation is enforced to zero by applying the proposed controller. Furthermore, by a suitable choice of this surface, the tracking error tends to zero asymptotically, in finite time and without any chattering problems.

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